Technical Note:
Using life table methods to calculate QALY losses from deaths: with application to COVID-19

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First the standard approach to estimating life-expectancy is outlined with a focus on conditional life-expectancy having reached a given age. We then demonstrate how this standard approach is easily adapted to adjust for both morbidity and mortality effects of comorbidity, to give quality adjusted life-expectancy, before applying discounting to give the potential discounted QALY loss associated with a death at any given age.

2.1 Standard life table approach to estimating life-expectancy

Life tables are produced nationally and show the numbers of people dying in one-year age bands across a population. We start by defining $q(x)$ as the probability of dying between ages x and $x+1$. From this we can calculate $l(x)$, for a reference population of 100,000, the number surviving to age $x \geq 1$ as:

$$l(x) = 100{,}000 \times \prod_{a=1}^{x} \{1 - q(a)\}$$

where $l(0) = 100{,}000$ by definition.

We now define $L(x)$ as the person-years lived between ages x and $x+1$ for $x \geq 1$:

$$L(x) = \frac{l(x) + l(x+1)}{2},$$

(assuming a uniform distribution of death during the year) and the total number of person-years lived above age x as:

$$T(x) = \sum_{u=x}^{\omega} L(u)$$

where ω is the upper bound of life-expectancy reported in the life table.

Now we calculate the life expectancy at age x as

$$LE(x) = \frac{T(x)}{l(x)}.$$

2.2 Adjusting for comorbidity, quality of life and time preference

Three steps to adjusting the standard method are outlined below in order to introduce: 1) the mortality impacts of comorbidity on life-expectancy; 2) quality of life adjustment to estimate QALYs; and 3) discounting.

Comorbidities can increase a subject's risk of death. In epidemiology, the standardized mortality ratio (SMR) summarizes how a given comorbidity can increase the risk of dying. However, applying SMR directly to the probability of death within a period would risk the probability exceeding one, especially for older ages. We therefore estimate the underlying instantaneous death rate, $d(x) = -ln\{1 - q(x)\}$, that corresponds to the per period death probability, $q(x)$, and apply an SMR parameter to this underlying rate. This gives the equation for the reference population surviving to age x, $1 \leq x < \omega$ to give:

$$l_s(x) = 100{,}000 \times \prod_{a=1}^{x} e^{-d(a) \cdot SMR}$$

with $L_s(x)$ the average of the adjacent as previously defined.

Next, we adjust for health-related quality of life by age. Standard population norm tables have been published for EQ-5D tariff values that can be used to adjust life-years to give QALYs for many different jurisdictions (Janssen B & Szende A, 2014). These tables give the population average quality of life tariff as a function of age x, $Q(x)$. Multiplying $L(x)$ by $Q(x)$ and an additional parameter to account for comorbidity impacts on quality of life, qCM, allows the calculation of quality-adjusted $T(x)$ and dividing by $l_s(x)$ gives the quality-adjusted life-expectancy (QALE) at age x:

$$QALE(x) = \frac{\sum_{u=x}^{\omega} L_s(u) \cdot Q(u) \cdot qCM.}{l_s(x)}$$

The final step in providing an estimate of QALYs lost associated with a premature death at age x is to apply a discount rate r to account for the relative value of life years experienced in the future relative to the present:

$$dQALY(x) = \frac{\sum_{u=x}^{\omega} L(u) \cdot Q(u) \cdot qCM \cdot (1 + r)^{-(u-x)}}{l_s(x)}.$$