A standard method for estimating the standard error of HRQoL values.

Giselle Abangma & Andrew Briggs
London School of Hygiene & Tropical Medicine
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This note outlines the use of a standard formula for estimating the standard error (SE) of a prediction from a regression equation and shows how this can be applied to the EQ5D value set.

Recall that for correlated random variables X_1 and X_2, the expectation of their linear combination is equal to the sum of their expectations (equation 1) and the variance is a function the sum of their variances plus twice their covariance of these two random variables (equation 2)[2]:

$$E(X_1 + X_2) = E(X_1) + \mathrm{E}(X_2) \quad (1)$$

$$Var(X_1 + X_2) = \mathrm{Var}(X_1) + Var(X_2) + 2Cov(X_1, X_2) \quad (2)$$

Adding constants, b_1 and b_2 to the linear combination, the expectation and variance are given as:

$$E(b_1X_1 + b_2X_2) = b_1E(X_1) + b_2E(X_2)$$

$$Var(b_1X_1 + b_2X_2) = {b_1}^2\, Var(X_1) + {b_2}^2\, Var(X_2) + 2b_1b_2Cov(X_1X_2)$$

We can expand this to a formula for n correlated random variables, the expectation and variance as:

$$E(b_1X_1 + b_2X_2 + \cdots + b_nX_n) = \sum_{i=1\ldots n} b_iE(X_i)$$

$$Var(b_1X_1 + \cdots + b_nX_n) = \sum_{i=1\ldots n} {b_i}^2\, Var(X_i) + \sum_{i<j} 2b_ib_jCov\left(X_iX_j\right)$$

Note: With regards to the EQ-5D (or any other preference-based instrument) where the descriptive system is fixed, the constants, b_1 and b_2, represent the dimensions of the descriptive system (i.e., the covariates in the regression model) and the random variables, X_1 and X_2, represent the coefficients which are estimated with uncertainty.

In matrix form, the variance of a linear combination of n corelated random variables (equation 10) is given as:

$$Var(b_1X_i + \cdots + b_nX_n) = (b_1 \ldots b_n)\Sigma \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = b'\Sigma b.$$

Where b' is the transpose of the covariate vector b and Σ denotes the covariance matrix of the coeficients. Using this formula, the SE of any health state value can be calculated – this is illustrated in the accompanying Excel tool for all 243 health states of the EQ5D.

References

1. Johnson, R. A. & Wichern, D. W. Applied Multivariate Statistical Analysis. (Prentice Hall, 2002).
2. Grami, Ali. Probability, Random Variables, Statistics, and Random Processes: Fundamentals & Applications. 1st edition. Hoboken, N.J: Wiley, 2020. Web.